AP CALCULUS BC SUMMER ASSIGNMENT

Going into AP calculus, there are certain skills that you have learned over the previous years that we assume you have retained. If you do not have these skills, you will find that you consistently miss problems next year, even if you understand the calculus concepts. Calculus is frustrating for students who are often tripped up by the algebra and not the calculus. We have a summer assignment that you will find on the school website (Academics→AP Program→Summer Reading) that is intended to help you brush up on your skills and possibly relearn these topics.

We assume that you have a solid foundation in precalculus. This foundation includes being able to solve equations, graph functions, work with algebraic expressions, and solve equations by factoring. These are skills that are used continually in AP Calculus.

Realize also that certain concepts are interrelated. Solving inequalities, for example, may require you to be an expert at identifying the domain of a function. Solving quadratic equations may involve techniques used in solving fractional equations.

This work is due the first day back to school in the fall.   You will be tested over this material. You need to get off to a good start, so print out the problems and spend some quality time on the packet this summer. Staple the pages and be sure your name appears on the first page. Work needs to be shown when needed. Also, do not rely on the calculator. Half of your AP exam next year will be taken without the calculator. Paper and pencil techniques only, please.

We recommend that you wait until July to begin the packet.   We want these techniques to be fresh in your mind in the fall. Also, do not wait until the last minute to complete the packet. Be sure to allow yourself enough time to review concepts and produce quality work.

If you have questions about any of these problems or the techniques used in solving them, contact Mr. Michael at the school website/email address. Have a good summer and see you in the fall!

Mr. Michael: chris_michael@gwinnett.k12.ga.us

Summer Assignment 2011 Name:__________________________

Section I: Algebra Review

Factoring

1. x^3+8

2. x^3-8

3. $27x^3-125y^3$

4. x^4+11x^2-80

5. $ac-cd+ab-bd$

6. $2x^2+50y^2-20xy$

7. $x^2+12x+36-9y^2$

8. $x^3-xy^2+x^2y-y^3$

9. $e^x(2x+1)^3+e^{2x}(2x+1)^2$

Simplifying Expressions

10. $\dfrac{\frac{1}{x}}{x-\frac{1}{2}}$

11. $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

12. $\dfrac{x-\frac{1}{2x}}{x+\frac{1}{3x}}$

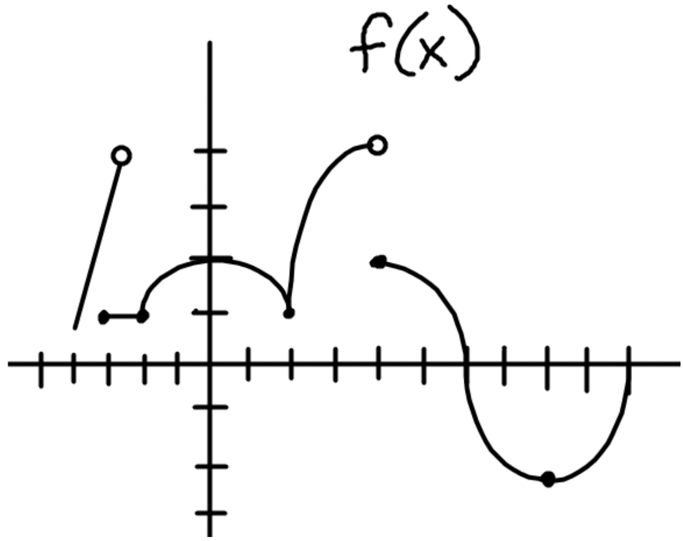
Rewrite the expression by multiplying by the conjugate

13. $\dfrac{x}{\sqrt{x+5}-\sqrt{5}}$

14. $\dfrac{\sqrt{2x+3}-\sqrt{3}}{x}$

Section II: Functions

Increasing/Decreasing



1. Determine the interval(s) over which f(x) is:

a. Increasing ___________________________

b. Decreasing __________________________

c. Constant ____________________________

d. Concave Down______________________

e. Linear ______________________________

f. Concave Up _________________________

g. What are the zeros of f? _______________

h. For what values of x is f(x) discontinuous? ______________________

Compositions

2. Let $f(x) = 3x^2$ and $g(x) = \dfrac{x-9}{x+1}$, find the following:

a. $f(g(x))$

b. $g(f(x))$

c. $f^{-1}(x)$

d. Domain, Range, and Zeros of f(x)

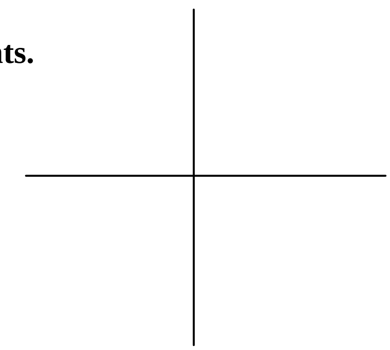
e. Domain, Range, and Zeros of g(x)

Find f^{-1} and verify that $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$.

3. $f\left(x\right) = 2x + 3$

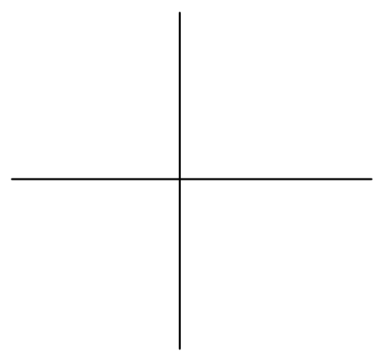
4. $f\left(x\right) = x^3 - 1$

Piecewise Functions: Graph then evaluate the function at the indicated points.



5. $f(x) = \begin{cases} 3x + 2, & x > 3 \\ -x + 4, & x \le 3 \end{cases}$

a. f(2) b. f(3) c. f(5)



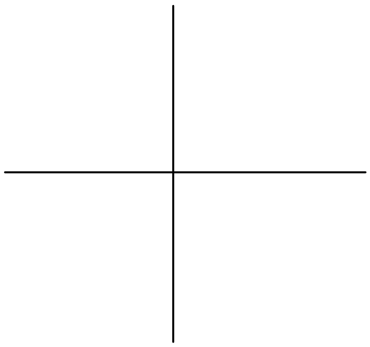
6. $f(x) = \begin{cases} x^2 - 1, & x < -2 \\ 4, & -2 \le x \le 1 \\ 3x + 1, & 1 < x < 3 \\ x^2 - 1, & x > 3 \end{cases}$

a. f(-3) b. f(-2) c. f(2) d. f(5) e) f(3)

Write the following absolute value expressions as piecewise functions and Graph

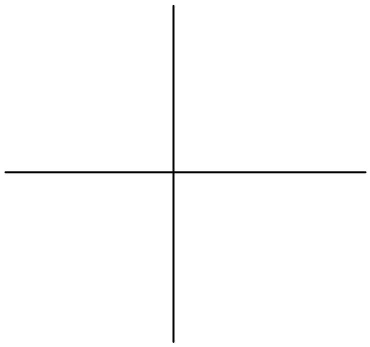
7. $y = |2x - 4|$

$f(x) = \begin{cases} \\ \end{cases}$



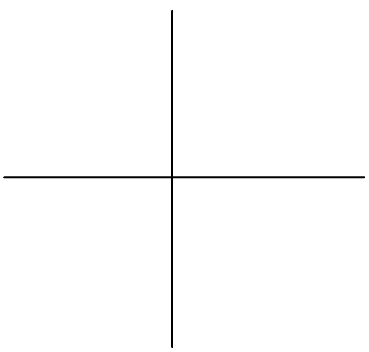
8. $y = |6 + 2x| + 1$

$f(x) = \begin{cases} \\ \end{cases}$



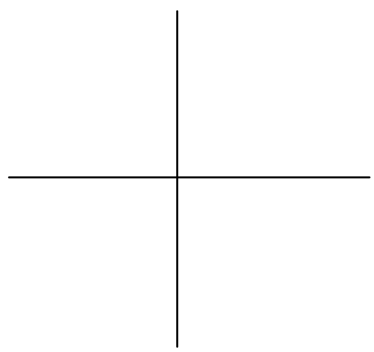
10. $y = |x^2 - 1|$

$f(x) = \begin{cases} \\ \end{cases}$



11. $y = |x^2 - 4x - 12|$

$f(x) = \begin{cases} \\ \end{cases}$



Even/Odd Functions

Show work to determine if the relation is even, odd, or neither.

12. $f(x) = 2x^2 - 7$

13. $f(x) = -4x^3 - 2x$

14. $f(x) = 4x^2 - 4x + 4$

15. $f(x) = x - \frac{1}{x}$

16. $f(x) = |x| - x^2 + 1$

17. $f(x) = \sin x + x$

Domains of Functions: Find the Domain of each.

18. $y = \frac{3x-2}{4x+1}$

19. $y = \frac{x^2-4}{2x+4}$

20. $y = \frac{x^2-5x-6}{x^2-3x-18}$

21. $y = \frac{2^{2-x}}{x}$

22. $y = \sqrt{x-3} - \sqrt{x+3}$

23. $y = \frac{\sqrt{2x-9}}{2x+9}$

Asymptotes

Find the equation of both Horizontal and Vertical Asymptotes for the following functions. Find the coordinates of any holes.

24. $y = \frac{x}{x-3}$

25. $y = \frac{x+4}{x^2-1}$

26. $y = \frac{x^2-2x+1}{x^2-3x-4}$

27. $y = \frac{x^2-9}{x^3-3x^2-18x}$

Section III. Solving Equations and Inequalities

Solving Equations- Solve the following for x:

1. $x^2 - 9 = 0$

2. $3x^2 + 7x + 3 = 0$

3. $x^2 - 5x - 24 = 0$

4. $\frac{2x+5}{3x+1} = 0$

5. $\sin x = \frac{1}{2}$

6. $\ln x = 3$

7. $\ln x = 1$

8. $\ln x = e$

9. $\cos^2 x = \frac{1}{4}$

10. $2\sin^2 x + \sin x = 1$

11. $\cos^2 x + 2\sin x = 3$ (Use identity)

12. $2\sin x \cos x = 0$

13. $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

14. $\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$

15. $x^4 - 9x^2 + 8 = 0$

16. $x - 10\sqrt{x} + 9 = 0$

Solving Inequalities: Solve and graph the solution

17. $|x-3| > 12$

18. $|x-3| \le 4$

19. $|10x+8| > 2$

20. $x^2 - 16 < 0$

21. $x^2 + 6x - 16 \le 0$

22. $x^2 - 3x \ge 10$

Section IV: Trigonometry

1. Evaluate without a calculator:

a. $\cos\pi$

b. $\sin\frac{\pi}{6}$

c. $\sec 210^{\circ}$

d. $\tan 90^{\circ}$

e. csc (-150)

f. $\csc\frac{3\pi}{2}$

g. $\cos 0$

h. $\sin^{-1}\frac{-1}{2}$

i. $\cos^{-1}\frac{-\sqrt{3}}{2}$

j. $\tan^{-1} 1$

k. $\arcsin 0$

l. $\tan^{-1} -\sqrt{3}$

m. $\sin\frac{2\pi}{3}$

n. $\sin^{-1}\frac{\sqrt{2}}{2}$

o. $\arctan 0$

Section V: Properties of Logarithms and Exponentials

1. Evaluate without a calculator

a. $\log_4 64$

b. $\log_3 \frac{1}{9}$

c. $\log 10$

d. $\ln e$

e. $\ln 1$

f. $\ln e^3$

g. $3^{\log_3 7}$

h. $4^{\log_4 \sin x}$

2. Let $f(x) = \ln x$ for all x > 0, and let $g(x) = x^2 - 4$ for all real x.
Let H be the composition of f with g, that is $H(x) = f(g(x))$.
Let K be the composition of g with f, that is $K(x) = g(f(x))$.

(a) Find the domain of H.

(b) Find the range of H.

(c) Find the domain of K.

(d) Find the range of K

(e) Find H(7)

Section VI: Lines

Find the equation of the line using point-slope form : $y - y_1 = m(x - x_1)$

1. (2, 3), (5, 6)

2. (12, 1), (5, 0)

3. (5, 5), (8, 5)

4. (-3, 7), (-3, 14)

5. (-11, -12), (-15, -4)

Using the above, write the equation of the lines that are parallel and perpendicular through the point (1, 1).

6. Parallel to # 1 — Perpendicular to # 1

7. Parallel to # 2 — Perpendicular to # 2

8. Parallel to # 3 — Perpendicular to # 3

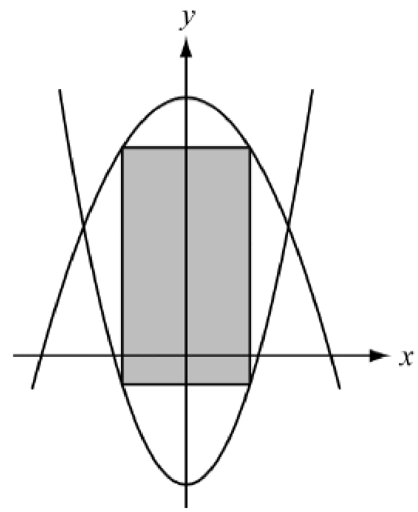
9. Parallel to # 4 — Perpendicular to # 4

10. Parallel to # 5 — Perpendicular to # 5

Section VII: Free Response Questions

Calculator Allowed

1. Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$ by answering the following:



a. Describe the width of the rectangle in terms of x

b. Describe the height of the rectangle in terms of f(x) & g(x)

c. Describe the height of the rectangle in terms of x

d. Write the function A(x) of the area of the rectangle in terms of the width and height found in parts a & c.

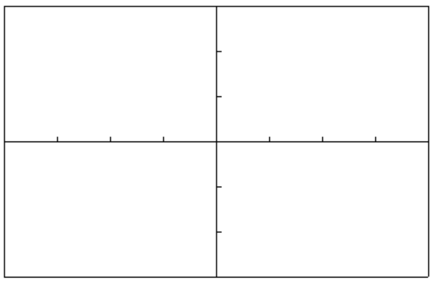
e. Find the maximum of A(x)

2. Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides.

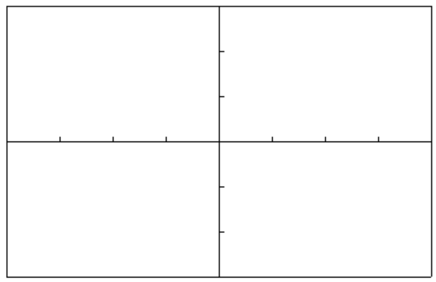
Section VIII: Parametric Equations

The mention of the curve $y = x^2$ should summon an immediate mental image of a parabola on the coordinate plane. The following parametric curve descriptions are related to the curve $y = x^2$, but perhaps do not evoke a mental image as quickly. Graph the following curves indicating direction for increasing values of t in the domain of each curve. Also indicate the value(s) of t corresponding to the domain endpoints and the point corresponding to $t = 0$, if any. (Reminder: to graph a parametric equation, you can make a table of t, x, and y values with t as the independent variable.)

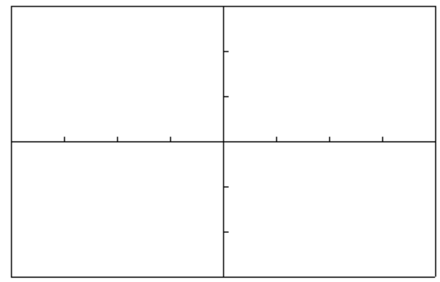
1. $x = t,\ y = t^2$



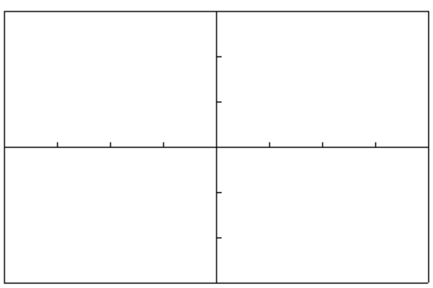
2. $x = -t,\ y = t^2$



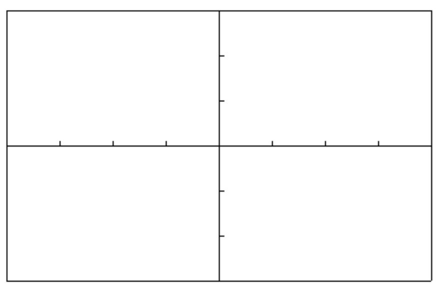
3. $x = t^2,\ y = t$



4. $x = t^2,\ y = t^4$

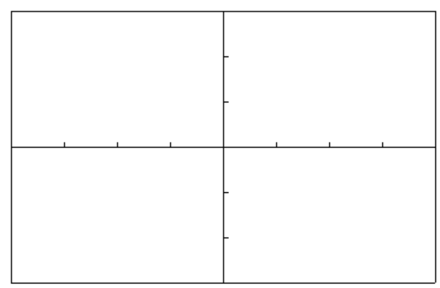


5. $x = \sin t,\ y = 1 - \cos^2 t$

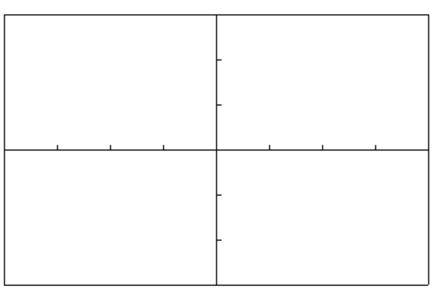


6. $x = \sec t,\ y = \sec^2 t$

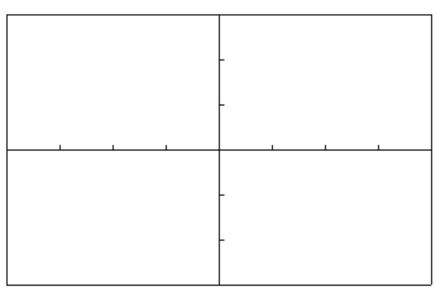
$0 \le t \le \pi,\ t \ne \frac{\pi}{2}$



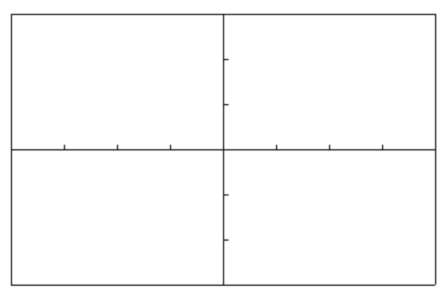
7. $x = t^2,\ y = |t|$



8. $x = e^t,\ y = e^{2t}$

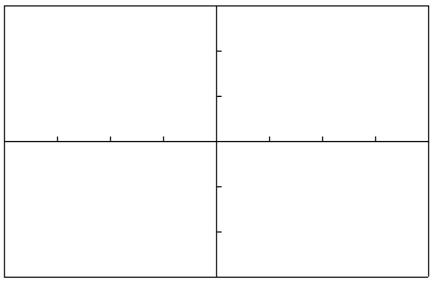


9. $x = \frac{1}{t},\ y = \frac{1}{t^2},\ t \ne 0$



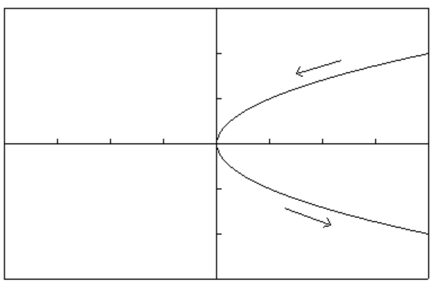
10. In parametric form a graph can easily be reflected over the line $y = x$. If the graph of $x = f(t),\ y = g(t)$ is reflected over the line $y = x$, the new graph is described by $x =$ ________, $y =$________. Which of the above curves (#1 – 9) is a reflection over the line $y = x$ of the graph of $y = x^2$? _________________

11. For exercise 8, find a corresponding parametric description of the reflection of the curve over the line $y = x$, then sketch the graph?

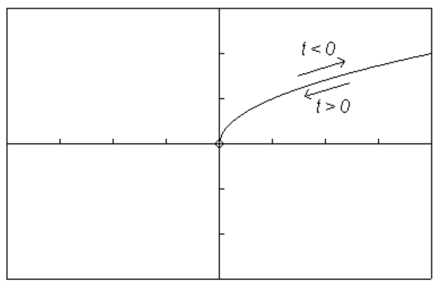


12. The following graphs of $y^2 = x$ or $y = \sqrt{x}$ are drawn indicating direction for increasing values of t. Provide a parametric description of each:

a.



b.

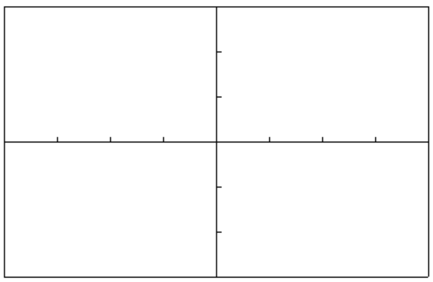


Section IX: Polar Graphing

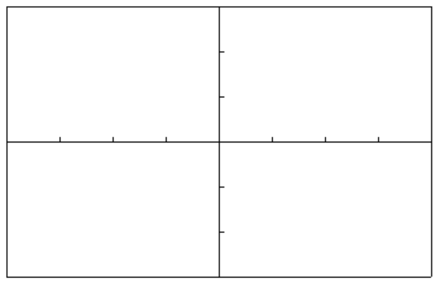
There are famous families of curves in the polar-coordinate system with polar equations that are easily recognizable. Review the following generalizations and complete the graphs as indicated. (Reminder: to graph a polar equation, you can make a table of r and θ values with θ as the independent variable.) You may use your calculator to check your work (MODE: Polar), but try to do as much as possible by hand.

1. Family of Lines – The rectangular form of a line $ax + by = c$ translates in polar form to $ar\cos\theta + br\sin\theta = c$. The following are often-used special cases of linear equations. Sketch each line.

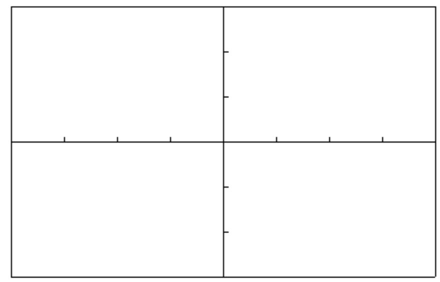
a. $r\cos\theta = 1$ or $r = \sec\theta$



b. $r\sin\theta = 1$ or $r = \csc\theta$

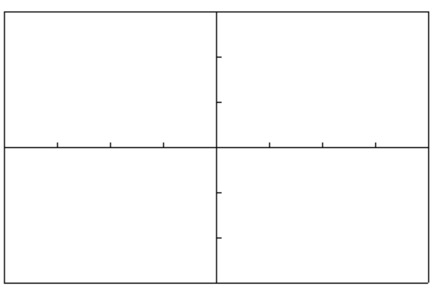


c. $\theta = 1$

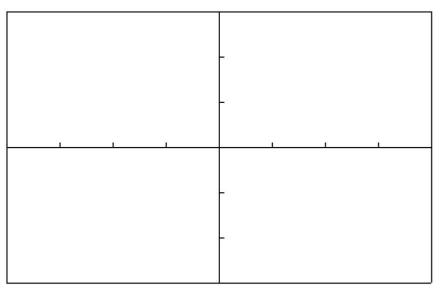


2. Family of Circles – The rectangular form of a circle $x^2 + y^2 = a^2$, with center at the origin, translates in polar form to $r = a$. If the center is not at the pole, then the following forms are common. Sketch each circle.

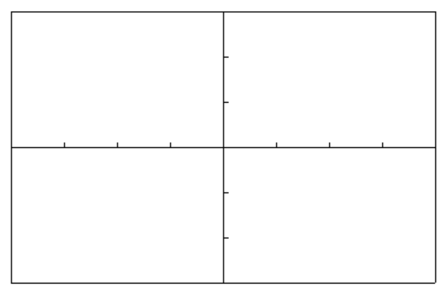
a. $r = 2\cos\theta$



b. $r = 2\sin\theta$

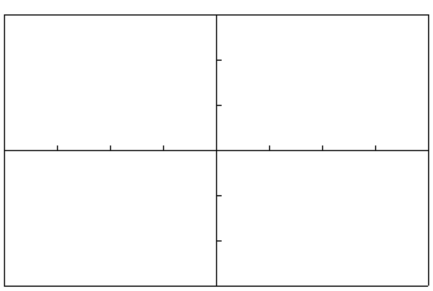


c. $r = 2\cos\theta + 3\sin\theta$

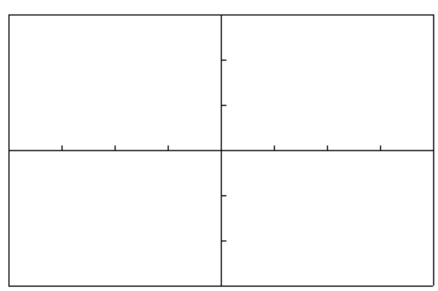


3. Family of Roses – A rose has no easily recognizable rectangular form but in polar form can be generally described as $r = a\cos(n\theta)$ or $r = a\sin(n\theta)$, where *n* is a natural number. If *n* is even, the rose has 2*n* petals and 4*n* lines of symmetry; if *n* is odd, the rose has *n* petals and *n* lines of symmetry. Sketch:

a. $r = 2\cos 3\theta$

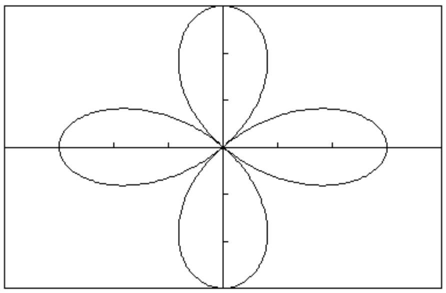


b. $r = -3\sin 2\theta$

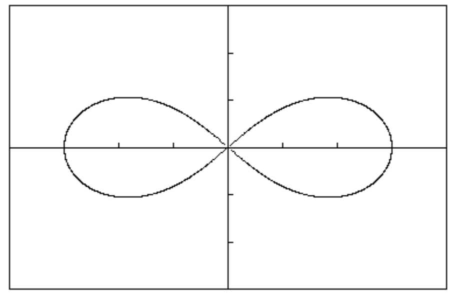


4. Family of Lemniscates – Polar form: $r^2 = a^2 \cos 2\theta$ or $r^2 = a^2 \sin 2\theta$
Note similarities to the four-petaled rose: $r = a\cos 2\theta$ or $r = a\sin 2\theta$. For example, $r = a\cos 2\theta$ would "lose" two petals on becoming $r^2 = a^2 \cos 2\theta$, namely, the petals with axis of symmetry $\theta = \frac{\pi}{2}$.

Actually, the shape of a lemnicate's petals is different, but the thought of a "rose connection" may assist in sketching graphs. Note that a graphing calculator is likely to "cut off" the portions of a lemnicate's petals near the pole, depending on the θ-interval chosen.

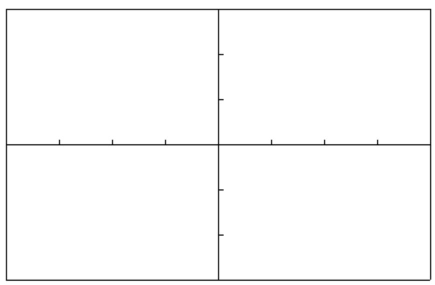


$r = 3\cos 2\theta$



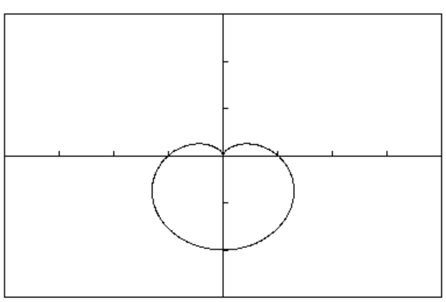
$r^2 = 9\cos 2\theta$

Sketch $r^2 = 4\sin 2\theta$

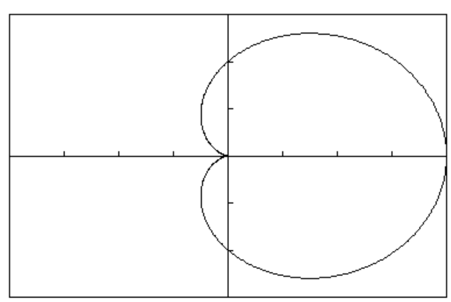


5. Family of Limaçons – Polar form: $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$
The shapes of limaçons depend on the values of *a* and *b* used in the equations above.
If $|a| = |b|$, then the limaçon includes the pole, and either $r \geq 0$ for the entire curve, or $r \leq 0$ for the entire curve.

Examples:



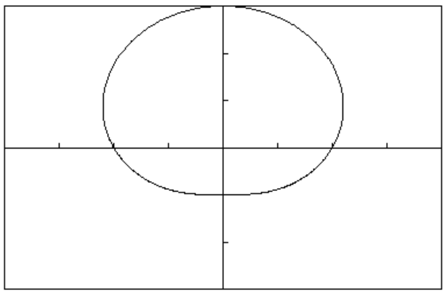
$r = 1 - \sin\theta$



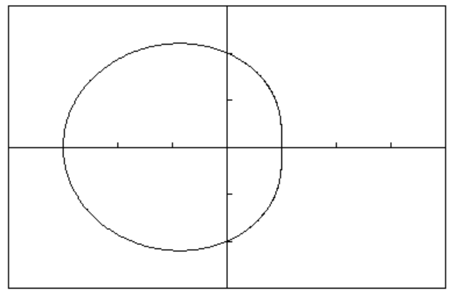
$r = -2 + 2\cos\theta$

If $|a| > |b|$, then either $r > 0$ for entire curve, or $r < 0$ for the entire curve. These limaçons do not include pole.

Examples:



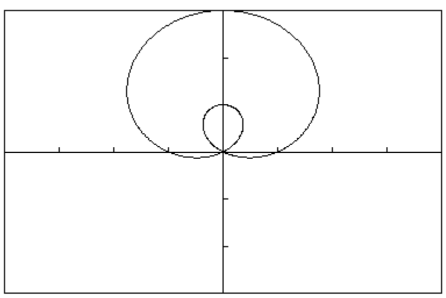
$r = 2 + \sin\theta$



$r = -2 - \cos\theta$

If $|a| < |b|$, then the limaçon involves both positive and negative values of r and the limaçon has an "inner loop".

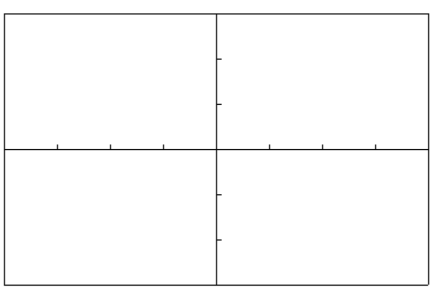
Examples:



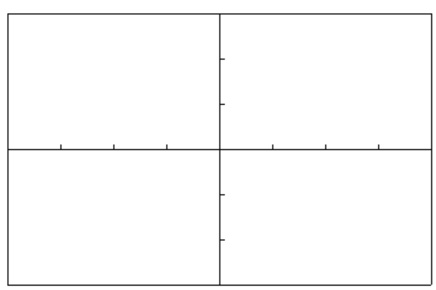
$r = 1 + 2\sin\theta$

Sketch:

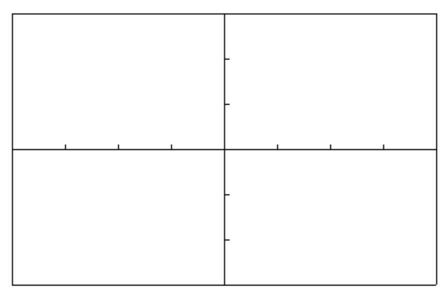
a. $r = \cos\theta - 1$



b. $r = 1 + 2\sin\theta$



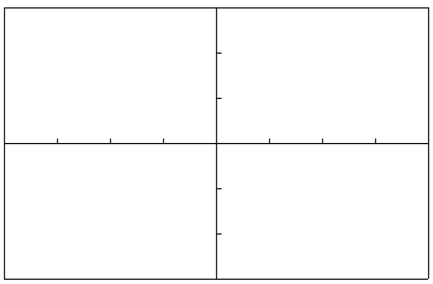
c. $r = 2 - \cos\theta$



6. The concept of equations in polar form is closely related to the concept of parametric equations. Any equation in the form $r = f(\theta)$ has the same graph as the parametric equations $x = f(\theta)\cos\theta$, $y = f(\theta)\sin\theta$.

For each parametrization below, sketch the graph of the parametrized curve and give the equation in polar form.

a.



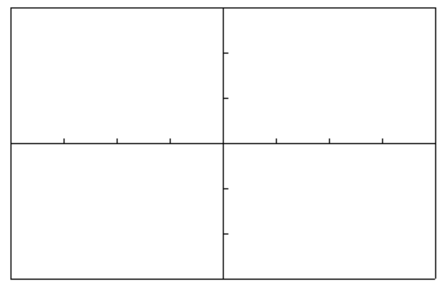
$x = (2 + \sin t)\cos t$

$y = (2 + \sin t)\sin t$

$0 \le t \le 2\pi$

Polar form: ________________

b.



$x = 2\cos t \sin 5t$

$y = 2\sin t \sin 5t$

$0 \le t \le 2\pi$

Polar form: ________________

Memorize the following Trig Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\sec^2\theta = 1 + \tan^2\theta$ $\qquad$ $\csc^2\theta = 1 + \cot^2\theta$

$\sin(-\theta) = -\sin\theta$ $\qquad$ $\cos(-\theta) = \cos\theta$ $\qquad$ $\tan(-\theta) = -\tan\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ $\qquad$ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\qquad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

Graph each of the following Parent Functions:

1. $f(x) = x$
2. $f(x) = x^2$
3. $f(x) = x^3$
4. $f(x) = |x|$
5. $f(x) = \sqrt{x}$
6. $f(x) = \dfrac{1}{x}$
7. $f(x) = \dfrac{1}{x^2}$
8. $f(x) = e^x$
9. $f(x) = \ln x$
10. $f(x) = \tan x$
11. $f(x) = \sin x$
12. $f(x) = \cos x$
13. $f(x) = \tan^{-1} x$
14. $f(x) = x^{\frac{2}{3}}$
15. $f(x) = \dfrac{1}{1+x^2}$
16. $f(x) = [\![x]\!]$
17. $f(x) = \sqrt{1-x^2}$
18. $f(x) = \dfrac{|x|}{x}$

Exponential and Logarithm Rules

$\ln e = 1$ $\qquad$ $\ln 1 = 0$ $\qquad$ $\ln 0 = \text{undefined}$

$\log_a a = 1$ $\qquad$ $\log_a 1 = 0$ $\qquad$ $\log_a 0 = \text{undefined}$

$\log a + \log b = \log ab$ $\qquad$ $\log a - \log b = \log\dfrac{a}{b}$ $\qquad$ $\log a^b = b\log a$

$a^{\log_a x} = x$ $\qquad$ $e^{\ln x} = x$ $\qquad$ $\log_a a^x = x$ $\qquad$ $\ln e^x = x$